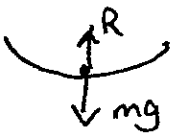
edexcel

January 2006
6682 Mechanics M6
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $r = a\sin 2\theta \Rightarrow \dot{r} = 2a\cos 2\theta . \omega$	M1 A1
	$v^2 = \dot{r}^2 + (r\dot{\theta})^2 = 4a^2\omega^2\cos^2 2\theta + a^2\omega^2\sin^2 2\theta$	M1
	$\theta = \pi/8 \quad v^2 = 4a^2\omega^2 . \frac{1}{2} + a^2\omega^2 . \frac{1}{2}$	M1
	$\Rightarrow v = \sqrt{\frac{5}{2}}\, a\omega$	A1 (5)
	(b) $Acc_T = r\ddot{\theta} + 2\dot{r}\dot{\theta}$; $\ddot{\theta} = 0 \Rightarrow Acc_T = 4a\omega^2\cos 2\theta$	M1
	$\theta = \pi/4$: $\cos 2\theta = 0 \quad Acc_T = 0$ (*)	M1 A1 (3) (8)
2.	(a) $m\ddot{s} = -mg\sin\psi \Rightarrow \ddot{s} = -\frac{g}{2a}s$ (*)	M1 A1 (2)
	(b) $\int \dot{s}\, d\dot{s} = -\frac{g}{2a}\int s\, ds$	
	$\frac{1}{2}\dot{s}^2 = -\frac{g}{4a}s^2 \ (+C)$	M1
	$(t=0) \quad s = \frac{3}{2}a, \ \dot{s} = 0 \Rightarrow C = \frac{9ga}{16}$	M1
	$s = 0 \quad \dot{s}^2 = \frac{9ga}{8} \Rightarrow \dot{s} = \frac{3}{2}\sqrt{\frac{ga}{2}}$ (*)	A1 (3)
	(c)  $R - mg = \frac{mv^2}{\rho}$	M1 A1
	$\rho = \frac{ds}{d\psi} = 2a\cos\psi$	B1
	Sub $\psi = 0, v = \frac{3}{2}\sqrt{\frac{ga}{2}}$: $R = mg + \frac{m}{2a} . \frac{9ga}{8}$	M1
	$= \frac{25mg}{16}$	A1 (5) (10)

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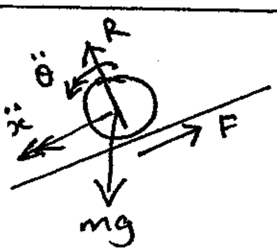
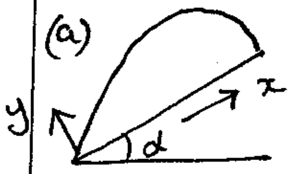
3.	MI of hoop about $P = ma^2 + ma^2 = 2ma^2$	M1 A1
	Moment of mom^m about P:	
	$mv.\frac{2}{3}a + ma^2.\omega = 2ma^2.\omega'$	M1 A3,2,1,0
	Rolling before $\Rightarrow v = a\omega$	B1
	Solve $\longrightarrow$ $\underline{\omega' = \frac{5}{6}\omega}$	M1 A1 (9)
4.		
	R(↖) $R = mg\cos\alpha$	B1
	Sliding $\Rightarrow F = \mu R$	B1
	$Fa = \frac{2}{5}ma^2\ddot{\theta}$ $(= \mu mga\cos\alpha)$	M1 A1
	R(↙) $m\ddot{x} = mg\sin\alpha - F$	M1 A1
	$\dot{x} = (g\sin\alpha - \mu g\cos\alpha)t, +V$	M1 A1, A1
	$\dot{\theta} = \frac{5\mu g\cos\alpha}{2a}t$	M1 A1
	Slips until $\dot{x} = a\dot{\theta}$, i.e. when	M1
	$(g\sin\alpha - \mu g\cos\alpha)t + V = \frac{5}{2}\mu g\cos\alpha\, t$	M1 A1
	$\Rightarrow t = \dfrac{2V}{g(7\mu\cos\alpha - 2\sin\alpha)}$ (*)	A1 (15)

Qu.5.

(a)



$\dot{x} = V\cos\theta - g\sin\alpha\, t$ M1

$= V\cos\theta - \frac{3}{5}gt$ A1

$y = V\sin\theta\, t - \frac{1}{2}g\cos\alpha\, t^2$ M1

$= V\sin\theta\, t - \frac{2}{5}gt^2$ A1

$y = 0,\ \dot{x} = 0$: from $\dot{x}$ $t = \dfrac{5V\cos\theta}{3g}$ M1 A1

Hence (y. y) : $V\sin\theta = \frac{2}{5}g \cdot \dfrac{5V\cos\theta}{3g}$ M1

$\Rightarrow \underline{\tan\theta = \frac{2}{3}}$ (*) A1 (8)

(b) $x = V\cos\theta\, t - \frac{3}{10}gt^2$ M1 A1

$\dot{x} = 0,\ t = 2 \Rightarrow 2 = \dfrac{5V\cos\theta}{3g} \Rightarrow V\cos\theta = \dfrac{6g}{5}$ M1

$\Rightarrow x = \frac{6g}{5} \times 2 - \frac{3}{10}g \times 4$ M1

$= 1.2g \simeq \underline{11.8\text{ m}}$ A1 (5)

(c) $V = \dfrac{6g}{5\cos\theta}, = \dfrac{6g}{5\cos(\arctan 2/3)}$ M1, M1

$\simeq \underline{14.1\text{ m s}^{-1}}$ A1 (3)

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Qu. 6	(a)	Trans. accel^n $= 0 \Rightarrow r^2\dot{\theta} = k$	M1
		$t=0,\ r=1,\ r\dot{\theta}=3 \Rightarrow k=3$	M1 A1
		Tens^n in string $= 4(r-1)$	B1
		Hence $2(\ddot{r} - r\dot{\theta}^2) = -4(r-1)$	M1 A1
		$r\dot{\theta}^2 = r.\frac{9}{r^4} = \frac{9}{r^3}$	M1
		$\Rightarrow \underline{\ddot{r} = \frac{9}{r^3} - 2r + 2}$ (*)	A1 (8)
	(b)	$\int \dot{r}\,d\dot{r} = \int \frac{9}{r^3} - 2r + 2\ dr$	
		$\frac{1}{2}\dot{r}^2 = -\frac{9}{2r^2} - r^2 + 2r\ (+C)$	M1 A1
		$t=0,\ r=1,\ \dot{r}=0 \Rightarrow C = \frac{7}{2}$	M1
		Hence $\underline{\dot{r}^2 = 7 - \frac{9}{r^2} - 2r^2 + 4r}$ (*)	A1 (4)
		$\dot{r}^2 = \frac{1}{r^2}(-2r^4 + 4r^3 + 7r^2 - 9)$	
		$= \frac{1}{r^2}(r-1)(-2r^3 + 2r^2 + 9r + 9)$	M1
		$= \frac{1}{r^2}(r-1)(3-r)(3 + 4r + 2r^2)$	M1 A1
		$3 + 4r + 2r^2$: "$b^2 - 4ac$" $= -8$	
		Hence no real roots – always > 0	M1
		Hence $(r-1)(3-r) \geqslant 0$	
		$\Rightarrow \underline{1 \leq r \leq 3}$ cso	A1 (5)
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